

Incentive Mechanisms for Hierarchical Spectrum Markets

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Abstract—In this paper, we study spectrum allocation mechanisms in hierarchical multi-layer markets which are expected to proliferate in the near future based on the current spectrum policy reform proposals. We consider a setting where a state agency sells spectrum channels to Primary Operators (POs) who subsequently resell them to Secondary Operators (SOs) through auctions. We show that these hierarchical markets do not result in a socially efficient spectrum allocation which is aimed by the agency, due to lack of coordination among the entities in different layers and the inherently selfish revenue-maximizing strategy of POs. In order to reconcile these opposing objectives, we propose an incentive mechanism which aligns the strategy and the actions of the POs with the objective of the agency, and thus leads to system performance improvement in terms of social welfare. This pricing-based scheme constitutes a method for hierarchical market regulation. A basic component of the proposed incentive mechanism is a novel auction scheme which enables POs to allocate their spectrum by balancing their derived revenue and the welfare of the SOs.

I. INTRODUCTION

A. Background and Motivation

Nowadays, it is common belief that the current coarse and static spectrum management policy creates a spectrum shortage. While this resource is expensive and scarce, significant amount of the reserved spectrum remains idle and unexploited by legitimate owners. A prominent proposed solution is the reform of the spectrum allocation policy and the deployment of dynamic spectrum (DS) markets, [1]. Spectrum should be allocated in a finer spatio-temporal scale to the interested buyers, the so-called primary operators (POs), [2] and more importantly, the POs should be able to lease their idle spectrum to secondary operators (SOs), [3], who serve fewer users in smaller areas. This method is expected to increase spectrum utilization and already several related business models exist in the market, [4]. However, the market-based solution is not a panacea and should be carefully applied.

These schemes will give rise to hierarchical spectrum markets where the spectrum will be allocated in two stages, i.e. from a state agency to the POs, and from each PO to the SOs. The objective of the agency, which we call hereafter *controller* (CO), is to allocate the spectrum efficiently, i.e. so as to maximize the aggregate social welfare from its use. However, this objective cannot be achieved by these markets because of the following two reasons: (i) the **coordination problem**, and the (ii) **objectives misalignment problem**. The

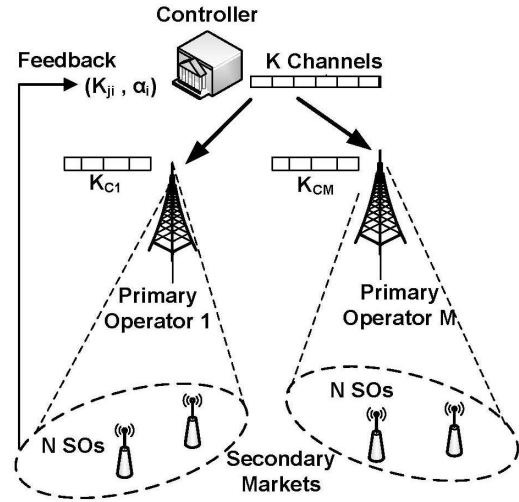


Fig. 1. The system consists of one Controller which has at his disposal K identical channels. There exist M primary operators which ask for spectrum. Each PO j acquires K_{Cj} channels and uses K_{j0} of them to satisfy the needs of his own users and resells K_{ji} channels to each SO i in the underneath secondary market. There are N SOs in the monopoly market under each PO which provide feedback to the CO for their needs and the decisions of the POs.

first problem emerges when the CO assigns the spectrum to the POs without considering the needs of the SOs (secondary demand). The second problem arises due to the inherently selfish behavior of POs who resell their spectrum in order to maximize their revenue. Clearly, this strategy contradicts the goal of the controller.

In this paper we study the spectrum allocation in these hierarchical markets and propose an *incentive mechanism* that enhances their performance by addressing the above two issues. The mechanism is deployed by the controller who acts as *regulator* and incentivizes the POs to redistribute their spectrum in a socially aware fashion. We consider a basic setting depicted in Figure 1, where each PO is a monopolist and has a certain clientele of SOs. Monopolies are expected to arise very often in these markets because the POs obtain the exclusive spectrum use rights for certain areas or because they collude and act effectively as one single seller. First, we analyze the performance of the unregulated hierarchical market, i.e. when there is no incentive mechanism, and we show that it

results in an undesirable equilibrium. The spectrum allocation from the CO to the POs and from the POs to the SOs is accomplished through auction-based mechanisms since there is lack of information about the spectrum demand. Namely, the CO uses an efficient auction such as the VCG auction, [5], while the POs employ an optimal auction, [6], which maximizes the expected revenue of the seller but induces efficiency loss, [7], [8].

Accordingly, we propose a pricing based incentive mechanism where the CO charges each PO in proportion to the inefficiency that is caused by his spectrum redistribution decisions. This way, the POs are induced to allocate their spectrum using a new auction scheme which produces less revenue for them but more welfare for the SOs. This is a *novel multi-item auction mechanism* where the objective of the auctioneer is a linear combination of his revenue and the valuations of the bidders. The balance between the objective of the POs and the SOs is tuned by a scalar parameter which is determined by the CO and reflects his regulation policy. Finally, we apply our mechanism to dynamic spectrum markets where the CO-POs and the PO-SOs interactions are realized in different time scale. Although in this case the coordination problem is inherently unsolvable, the proposed scheme still improves the performance of the market by aligning the decisions of the POs with the objective of the CO.

B. Related Work and Contribution

Optimal auctions were introduced by Myerson, [6] for single item allocation and extended later for multiple items, [9], or divisible resource, [8]. They maximize the expected revenue of the seller but are inefficient, [5], [7]. The interaction of primary and secondary operators is usually modeled as a monopoly market. For example, in [10] the authors consider a setting where each primary license holder sells his idle spectrum channels to a set of secondary users and show that the optimal auction yields higher profit but results in inefficient allocation. A similar monopolistic setting is considered in [11] and [12]. In [13], a multiple-item optimal auction is used by a primary service provider to allocate his channels to a set of secondary service providers while satisfying at the same time his own needs. It can be argued that even in oligopoly spectrum markets is highly probable that the POs - SOs interaction will result in spectrum allocation that is not efficient from the perspective of the controller, [14]. All these works analyze exclusively the primary - secondary operators interactions without taking into account the hierarchical structure of the spectrum markets.

This hierarchical aspect is studied in [15] where the authors consider a multi-level spectrum market and present a mechanism to match the demand and the spectrum supply of the interrelated spectrum markets in the different layers. Similar models have been considered in [16] and [17] where the buyers demand is considered known. Coordination problems have been also studied for bandwidth allocation in wireline networks, [18]. However, in these studies there is no misalignment among the objectives of the various entities (operators, users, etc) since they all maximize the revenue or the efficiency

of the allocation. *On the contrary, in the setting we study the entities have conflicting interests and the incentive mechanism we present achieves their alignment.* Our work is inspired by the sponsored search (keyword) auction mechanisms, [19], which assign the search engines advertising slots by taking into account the feedback from the *clickers*. Similar concepts can be used for the allocation of spectrum as we suggested in [20]. Here, we take a further step towards this direction by giving a detailed methodology.

In summary, the contributions of this paper are the following: (1) we analyze the hierarchical spectrum allocation and show that it is inefficient, (2) we present an incentive mechanism that motivates the POs to increase the efficiency of their spectrum redistribution, (3) we introduce the β -optimal auction which achieves a balance between the revenue of the seller (optimality) and the welfare of the buyers (efficiency). This is a new mechanism that can be used also for the allocation of similar communication resources (bandwidth, transmission power, etc), (4) we apply our mechanism to dynamic markets where the CO-POs and the PO-SOs interactions are realized in different time scales and we show that it improves their efficiency. To our knowledge, this is the first work that analytically studies the efficiency of the hierarchical spectrum markets and introduces a mechanism for their performance improvement.

The rest of the paper is organized as follows. In Section II we introduce the system model and in Section III we analyze the hierarchical spectrum allocation without the intervention of the controller. This analysis helps us to describe the incentive mechanism and assess its efficacy in Section IV. Finally in Section V we apply our mechanism to more dynamic spectrum markets. We present our numerical study in section VI and conclude in Section VII.

II. SYSTEM MODEL

We consider a three-layer hierarchical spectrum market with one *controller* (CO) on top of the hierarchy, a set $\mathcal{M} = \{1, 2, \dots, M\}$ of *primary operators* (POs) in the second layer and a set $\mathcal{N} = \{1, 2, \dots, N\}$ of *secondary operators* (SOs) that lie in the third layer under each PO, as it is shown in Figure 1. There exists a set $\mathcal{K} = \{1, 2, \dots, K\}$ of identical spectrum channels managed initially by the CO. The controller allocates the channels to the M primary operators and accordingly each PO redistributes the channels he acquired among himself and the N SOs that lie in his secondary market. The objective of the POs is to incur maximum revenue from reselling the spectrum while satisfying their own needs.

The perceived utility of each operator, PO or SO, for acquiring a channel is represented by a scalar value. Following the law of diminishing marginal returns, [14], we consider that each additional channel has smaller value/benefit for the operator. Different operators may have different spectrum needs and hence different channel valuations. For example, an operator with many clients will have very high channel valuations. Also, the POs are expected to have in general higher valuations than the SOs since they serve more users. We summarize these different characteristics of the operators

with a real-valued parameter which we call the *type* of the operator, [5], [13]. Notice that, our system model is general and satisfies the basic assumptions and requirements of many different settings, [11], [15], [16], [18].

Secondary Operators: In detail, the perceived utility of the i^{th} SO for the k^{th} channel is $U_k(\alpha_i) \in \mathcal{R}^+$ which is assumed to be positive, monotonically increasing and differentiable function of parameter α_i . This is the *type* of the SO and represents his spectrum needs. The types of the SOs in every secondary market $\alpha = (\alpha_i : i \in \mathcal{N})$ are mutually independent random variables, $\alpha_i \in \mathcal{A} = (0, A_{max})$, $A_{max} \in \mathcal{R}^+$, drawn from the same distribution function $F(\cdot)$ with finite density $f(\cdot)$ on \mathcal{A} . We assume that it holds: $U_1(\alpha_i) \geq U_2(\alpha_i) \geq \dots \geq U_K(\alpha_i) \geq 0$, for each $\alpha_i \in \mathcal{A}$, $i \in \mathcal{N}$. The SO i pays for the assigned channels an amount of money that is determined by the PO.

Primary Operators: Each j^{th} PO receives K_{cj} channels from the CO at a cost of $Q(K_{cj})$ monetary units and decides how many he will reserve for his own needs, K_{j0} , and how many he will allocate to each one of the N SOs at his secondary market, $\mathbf{K}_j = (K_{ji} : i \in \mathcal{N})$. We assume that the valuation of the j^{th} PO for acquiring the k^{th} channel is $V_k(p_j) \in \mathcal{R}^+$ which belongs to a known family of functions $V_k(\cdot)$ and is parameterized by the private variable $p \in \mathcal{P} = (0, P_{max})$, $P_{max} \in \mathcal{R}^+$. In analogy with α , p is the type of the PO and models his spectrum needs. The valuation functions are considered positive, monotonically increasing and continuously differentiable w.r.t. the type p_j and we assume that it is: $V_1(p_j) \geq \dots \geq V_K(p_j) \geq 0$. The benefit of the PO from reselling his spectrum is given by the revenue component $H(\mathbf{K}_j)$ which depends on the number of leased channels. We define the combined valuation - revenue objective of each PO $j \in \mathcal{M}$ as follows:

$$\hat{V}(p_j, K_{j0}, \mathbf{K}_j) = \sum_{k=1}^{K_{j0}} V_k(p_j) + H(\mathbf{K}_j) \quad (1)$$

Controller: The goal of the controller is to increase the spectrum utilization while ensuring the viability of the secondary markets. Therefore he acts as regulator and deploys an incentive mechanism to induce a channel allocation that maximizes a balanced sum of the POs' combined objectives and the valuations of the SOs:

$$C(\beta) = \sum_{j=1}^M [\hat{V}(p_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^N \sum_{k=1}^{K_{ji}} U_k(\alpha_i)] \quad (2)$$

where $\beta \in \mathcal{R}^+$ is defined by the CO and determines this balance. Apparently, as β increases, the allocation of spectrum will favor the SOs. Notice that the objective of the CO incorporates both the channel valuation of the POs and their revenue components, since the latter are the their motivation for reallocating the spectrum.

III. UNREGULATED HIERARCHICAL SPECTRUM ALLOCATION

We begin our study with the unregulated hierarchical spectrum allocation. In this case, the CO does not take into account

the secondary demand when he allocates the channels to the POs. The latter, may also be oblivious to the secondary demand at the moment they ask for spectrum, or they can make an early conjecture for the SOs needs or even they can be aware of the exact secondary demand. For all these scenarios, we show that the unregulated spectrum allocation induces efficiency loss. The model and analysis of this section is used in the sequel to introduce our mechanism and assess its efficacy.

A. First Stage: POs - CO Interaction

In the first stage, the POs ask for spectrum and the controller determines the channel distribution, $\mathbf{K}_c = (K_{cj} : j \in \mathcal{M})$ and the payment $Q(K_{cj})$. We assume that the CO knows the family of the valuation functions of the POs, $V_k(p)$, $k \in \mathcal{K}$ but not their exact types, [5], [19]. Therefore the CO runs a Vickrey-Clarke-Groove (VCG) auction, which is the most prominent efficient auction [5], in order to elicit this information. Every PO $j \in \mathcal{M}$ submits a scalar bid, $r_j \in \mathcal{P}$, in order to declare his type. The CO collects these bids, $\mathbf{r} = (r_j : j \in \mathcal{M})$, and finds the channel allocation that maximizes the total valuation of all the POs by solving the **CO Spectrum Allocation Problem**, (\mathbf{P}_{co}):

$$\max_{\mathbf{K}_c} \sum_{j=1}^M \sum_{k=1}^{K_{cj}} V_k(r_j) \quad (3)$$

s.t.

$$\sum_{j=1}^M K_{cj} \leq K, \quad K_{cj} \in \{0, 1, 2, \dots, K\} \quad (4)$$

One simple method to find the solution, \mathbf{K}_c^* , of problem \mathbf{P}_{co} , is to sort the valuations of the POs $V_k(r_j)$, $j \in \mathcal{M}$, $k \in \mathcal{K}$, in decreasing order and allocate the channels to the primary operators with the K highest valuations. Apparently, the number of channels the j^{th} PO receives depends both on his own bid r_j and the bids of the other POs, $\mathbf{r}_{-j} = (r_m : m \in \mathcal{M} \setminus \{j\})$, i.e. $K_{cj}(r_j, \mathbf{r}_{-j})$.

The payment imposed to each PO, according to the VCG payment rule [5], is equal to the externality he creates to the other POs:

$$Q(r_j, \mathbf{r}_{-j}) = \sum_{m \neq j}^M \sum_{k=1}^{\tilde{K}_{cm}^*} V_k(r_m) - \sum_{m \neq j}^M \sum_{k=1}^{K_{cm}^*} V_k(r_m) \quad (5)$$

where K_{cm}^* is the number of channels allocated to each PO $m \in \mathcal{M} \setminus \{j\}$ according to the solution of problem (\mathbf{P}_{co}) and \tilde{K}_{cm}^* the respective number when the j^{th} PO does not participate in the auction, i.e. $r_j = 0$.

Let us assume now that the POs bid without knowing the secondary demand. In this case, they consider only the benefit from using the acquired channels for their own needs, $K_{j0} = K_{cj}$, and hence they determine their bid by solving the **PO Bidding Problem**, (\mathbf{P}_{po}^b):

$$r_j^* = \arg \max_{r_j} \left\{ \sum_{k=1}^{K_{cj}(r_j, \mathbf{r}_{-j})} V_k(p_j) - Q(r_j, \mathbf{r}_{-j}) \right\} \quad (6)$$

Since VCG auctions are incentive compatible, [5], the POs will reveal their actual types, $r_j^* = p_j$, $\forall j \in \mathcal{M}$. However, if POs

are aware of the secondary demand (or can make a conjecture) then they will bid so as to maximize their combined valuation - revenue objective:

$$r_j^* = \arg \max_{r_j} \{\hat{V}(p_j, K_{j0}, \mathbf{K}_j) - Q(r_j, r_{-j})\} \quad (7)$$

Apparently, if the CO still uses the payment rule given by eq. (5) then the auction in this stage is not truthful anymore.

B. Second Stage: SOs - PO Interaction

In the second stage of the hierarchical spectrum allocation, each $j \in \mathcal{M}$ PO finds the optimal allocation, $(\mathbf{K}_j^*, K_{j0}^*)$, of his K_{cj} channels that maximizes his combined objective, eq. (1). This is given by the solution of the **PO Spectrum Allocation Problem**, (\mathbf{P}_{po}) :

$$\max_{\mathbf{K}_j, K_{j0}} \hat{V}(p_j, K_{j0}, \mathbf{K}_j) \quad (8)$$

s.t.

$$K_{j0} + \sum_{i=1}^N K_{ji} \leq K_{cj}, K_{ji}, K_{j0} \in \{0, 1, 2, \dots, K_{cj}\} \quad (9)$$

We assume that after receiving his spectrum from the CO, each PO obtains only partial information about the underneath secondary market. Namely, he learns the family of the SOs functions $U_k(\alpha)$, $k \in \mathcal{K}$, and the types distribution function $F(\cdot)$ but not the actual SOs types. To elicit this missing information the PO runs an *optimal auction* where each one of the N SOs submits a bid, $b_i \in \mathcal{A}$ in order to declare his type α_i . The PO collects the bids, $\mathbf{b} = (b_i : i \in \mathcal{N})$, and determines the allocated spectrum and the respective payment for each bidder. Here, the seller (PO) is also interested in the auctioned items and hence he compares his possible revenue from selling a channel with the valuation for using it, $V_k(\cdot)$ before he decides if he will allocate it to a SO or reserve it, [6],[13].

The maximization of the expected revenue, (\mathbf{P}_{po}) , can be transformed to a deterministic channel allocation problem. Let us first define the additional expected revenue the PO incurs for assigning the k^{th} channel to the i^{th} SO. In auction theory, [9], this is known as the *contribution* of the bidder and is defined as:

$$\pi_k(b_i) = U_k(b_i) - \frac{dU_k(\alpha)}{d\alpha} \Big|_{\alpha=b_i} \frac{1 - F(b_i)}{f(b_i)} \quad (10)$$

where $F(\cdot)$ and $f(\cdot)$ are the cdf and pdf of the SOs. If these contributions are monotonically strictly increasing in the SOs types and decreasing in the number of channels, then they satisfy the so-called *regularity conditions*, [9], and the auction problem \mathbf{P}_{po} is called *regular*. In this case the channel allocation that maximizes the combined objective of the j^{th} PO can be easily derived using the following deterministic allocation and payment rules.

1) *PO Optimal Auction Allocation Rule*: The auctioneer (PO j) calculates the contributions $\pi_k(b_i)$ of each SO $i \in \mathcal{N}$ for all the auctioned channels, $k = 1, \dots, K_{cj}$, and selects the K_{cj} highest of them. In the sequel, he compares these K_{cj} contributions with his own valuations for the channels

and constructs the contribution-valuation vector X_j which has K_{cj} elements in decreasing order:

$$X_j = (x_{(l)} : x_{(l)} > x_{(l+1)}, l = 1, \dots, K_{cj}) \quad (11)$$

Then, the PO simply assigns each channel $l = 1, \dots, K_{cj}$ to the respective i^{th} SO if $x_{(l)} = \pi_k(b_i)$ or he reserves it for himself if $x_{(l)} = V_k(p_j)$. For example, for a PO with 4 channels and two SOs bidders, a possible instance of X_j is $X_j = (V_1(p_j), \pi_1(b_1), \pi_1(b_2), V_2(p_j))$ which means that the PO will reserve 2 channels for himself and assign one to each SO.

2) *PO Optimal Auction Payment Rule*: The price that each SO i pays for receiving the k^{th} spectrum channel depends on the bids submitted by all the other SOs, $b_{-i} = (b_n : n \in \mathcal{N} \setminus \{i\})$. Namely, let us denote with $z_k(b_{-i})$ the minimum bid that the i^{th} SO has to submit in order to acquire the k^{th} channel, [9]:

$$z_k(b_{-i}) = \inf\{\hat{\alpha}_i \in \mathcal{A} : \pi_k(\hat{\alpha}_i) \geq \max\{0, x_{(K_{cj}+1)}\}\} \quad (12)$$

This means that in order to get the k^{th} item the i^{th} SO has simply to submit a bid high enough to draft his contribution within the first K_{cj} elements of X_j . The actual charged price for each channel is equal to his valuation had he a type equal to this minimum bid. Hence the aggregate payment for the SO is:

$$h(b_i, b_{-i}) = \sum_{k=1}^{K_{ji}(b_i, b_{-i})} U_k(z_k(b_{-i})) \quad (13)$$

This payment rule is an extension of the original rule introduced in [6] and [9] and has been used also for the case that the seller has valuation for the auctioned items in [13]. Hence, each SO $i \in \mathcal{N}$ bids according to the **SO Bidding Problem**, (\mathbf{P}_{so}) :

$$b_i^* = \arg \max_{b_i} \left\{ \sum_{k=1}^{K_{ji}(b_i, b_{-i})} U_k(\alpha_i) - h(b_{-i}) \right\} \quad (14)$$

Due to the payment and the respective monotonic allocation rule, the auction mechanism is incentive compatible and individual rational, [10], [19], hence $b_i^* = \alpha_i$, $\forall i \in \mathcal{N}$.

C. Inefficiency of the Unregulated Hierarchical Scheme

The first reason that renders inefficient this hierarchical allocation is the **coordination problem**: the CO allocates the channels to the POs by solving \mathbf{P}_{co} without considering the demand of the SOs. In case the POs are also unaware of the secondary demand the auction in the first stage is truthful and efficient wrt the POs needs but not wrt to the POs - SOs joint spectrum requirements. Namely, the CO may allocate more channels to a PO who is going to encounter smaller secondary demand than another PO. Now assume that the POs receive the bids of the SOs or make an early conjecture about the secondary demand before they ask CO for spectrum. In this case, if the CO is still unaware of the SOs needs, the hierarchical spectrum allocation is even more inefficient since the POs will bid in order to maximize their combined objective, according to eq. (7), and not their valuations, eq. (6). This means that in the first auction, the seller (CO) and

the bidders (POs) will use different valuation functions for determining the prices and the bids respectively. Therefore, the auction will not be incentive compatible anymore, i.e. $r_j \neq p_j$, $j \in \mathcal{M}$.

At the same time, and independently of the coordination problem, there exists the **objectives misalignment problem**. The CO and the POs have different goals and the revenue maximizing auction that is organized by the latter incurs efficiency loss, [5]. In the setting we study, this means that a PO may reserve a channel for himself while there exist SOs with higher valuation for it. Finally, notice that even if both the CO and the POs are aware of the exact secondary demand, still the problem of their objectives misalignment exists and renders inefficient the hierarchical spectrum allocation.

IV. REGULATED HIERARCHICAL SPECTRUM ALLOCATION

In this section we build upon the previous analysis and introduce our incentive mechanism. First, we explain the basic idea of the mechanism and the difficulties that the controller encounters in applying it. Next we introduce the β -optimal auction which is required in order to enable the POs to balance their revenue and the efficiency of their spectrum redistribution. Finally, we discuss the efficacy of the mechanism and its requirements.

A. Incentive Mechanism \mathcal{M}_R

The goal of the controller is to induce the channel allocation $\{K_{j0}^*, \mathbf{K}_j^*\}$ for each PO $j \in \mathcal{M}$ and the respective secondary market that maximizes his objective $C(\beta)$, given by eq. (2). This allocation stems from the solution of the **CO Balanced Spectrum Allocation Problem**, ($\mathbf{P}_{\text{co}}^{\text{bal}}$):

$$\max_{\{\mathbf{K}_{j0}, \mathbf{K}_j\}} \sum_{j=1}^M \left[\hat{V}(p_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^N \sum_{k=1}^{K_{ji}} U_k(\alpha_i) \right] \quad (15)$$

s.t.

$$\sum_{j=1}^M [K_{j0} + \sum_{i=1}^N K_{ji}] \leq K_c, K_{j0}, K_{ji} \in \{0, 1, 2, \dots, K_c\} \quad (16)$$

parameter $\beta \in \mathbb{R}^+$ is determined by the CO and defines implicitly the revenue of the POs and the welfare of the SOs.

The difficulties the controller encounters to achieve his goal, are three: **(i)** the CO is not aware of the types of the POs, p_j , $j \in \mathcal{M}$, **(ii)** he does not know the types of the SOs in each secondary market, a_i , $i \in \mathcal{N}$, and **(iii)** he cannot directly dictate the POs how to redistribute their channels. In economic terms, conditions (i) and (ii) capture the *hidden information asymmetry*, [14], of the spectrum market which means that the controller is not aware of the actual needs of the operators. Similarly, condition (iii) describes the *hidden action asymmetry*, which exists in the market because the CO is not aware of the actions of the POs. The introduced incentive mechanism, which we call *Mechanism \mathcal{M}_R* , eliminates these asymmetries and achieves the desirable spectrum allocation.

The proposed scheme is based on pricing and the underlying idea is that the controller creates a coupling between the

spectrum allocation decisions of the POs and their cost for acquiring the spectrum in order to bias their revenue maximizing strategy. Namely, we assume that the CO reimburses the j^{th} PO with the following price:

$$L_j(\alpha, \mathbf{K}_j, \beta) = \beta \sum_{i=1}^N \sum_{k=1}^{K_{ji}} U_k(\alpha_i) \quad (17)$$

This modifies the PO's objective function as follows:

$$\hat{V}_R(p_j, K_{j0}, \mathbf{K}_j, \beta) = \hat{V}(p_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^N \sum_{k=1}^{K_{ji}} U_k(\alpha_i) \quad (18)$$

$\hat{V}_R(\cdot)$ is the regulated new combined objective of each PO which depends on parameter β and is aligned with the balanced objective of the CO, eq. (2).

B. The β -Optimal Auction Mechanism

Each PO maximizes $\hat{V}_R(\cdot)$ by solving a new allocation problem $\mathbf{P}_{\text{po}}^\beta$ which differs from the respective \mathbf{P}_{po} problem in the objective function that is given now by eq. (18). Since the types of the SOs are unknown, the primary operator runs again an auction to elicit this hidden information. Nevertheless, this is neither an efficient nor an optimal auction and hence he cannot employ any of the known auction schemes. To address this problem, we introduce a new multi-item auction mechanism, the β -**optimal** auction, which ensures the maximization of the balanced objective defined in eq. (18). This mechanism is similar to the optimal auction discussed in section III-B with the difference that the allocation rule is biased by parameter β . This modification affects the channels allocation and results in reduced revenue for the auctioneer but improved allocation efficiency. The combination of optimal and efficient auctions has been also suggested in [21] for single item allocation where the authors proposed an efficient auction with a lower bound on the seller's revenue.

Let us now explain the rationale and machinery of the β -optimal auction. First we define the β -contribution for each SO $i \in \mathcal{N}$ under a certain PO $j \in \mathcal{M}$, as follows:

$$\pi_k^\beta(b_i) = (1 + \beta)U_k(b_i) - \frac{dU_k(\alpha)}{d\alpha} \Big|_{\alpha=b_i} \frac{1 - F(b_i)}{f(b_i)} \quad (19)$$

Since $\beta \geq 0$ it will be $\pi_k^\beta(\alpha_i) \geq \pi_k(\alpha_i)$ for all the SOs and all the channels. Moreover, notice that if the initial contributions satisfy the *regularity* conditions, [9], then the β -contributions will also satisfy them and hence problem $\mathbf{P}_{\text{po}}^\beta$ will be regular. Therefore, we can again derive deterministic channel allocation and payment rules.

β -Optimal Auction Allocation Rule: Similarly to the allocation rule of the optimal auction, the j^{th} PO calculates the $\pi_k^\beta(b_i)$ for all the SOs $i \in \mathcal{N}$ and all the channels $k = 1, \dots, K_{cj}$ and compares them with his own valuations in order to construct the contribution-valuation vector X_j^β :

$$X_j^\beta = (x_{(l)}^\beta : x_{(l)}^\beta > x_{(l+1)}^\beta, l = 1, \dots, K_{cj}^\beta) \quad (20)$$

Using X_j^β the PO allocates his channels to the respective K_{cj} highest contributions and valuations. The resulting channel

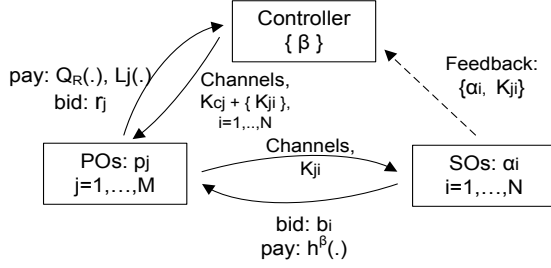


Fig. 2. The machinery of incentive mechanism \mathcal{M}_R . The circulated information, bids and channel allocation among the SOs, POs and the Controller is depicted. The feedback can be provided from the SOs to the CO directly, or be inferred using other means.

allocation $(K_{j0}^\beta, \mathbf{K}_j^\beta)$ solves problem \mathbf{P}_{po}^β and maximizes the new regulated objective $\hat{V}_R(p_j, K_{j0}^\beta, \mathbf{K}_j^\beta, \beta)$. Again, this allocation rule is monotone increasing in the types of the SOs.

β -Optimal Auction Payment Rule: The payment rule changes in order to comply with the new allocation rule. Namely, the minimum bid that the i^{th} SO needs to submit in order to acquire the k^{th} channel is:

$$z_k^\beta(b_{-i}) = \inf\{\hat{\alpha}_i \in \mathcal{A} : \pi_k^\beta(\hat{\alpha}_i) \geq \max\{0, x_{(K_{cj}^\beta+1)}^\beta\}\} \quad (21)$$

and, similarly to the previous mechanism, the total payment for this SO is :

$$h^\beta(b_i, b_{-i}) = \sum_{k=1}^{K_{ji}^\beta(b_i, b_{-i})} U_k(z_k^\beta(b_{-i})) \quad (22)$$

Under this new auction mechanism, each SO $i \in \mathcal{N}$ selects his bid so as to maximize his payoff, (**SO β -Bidding Problem, \mathbf{P}_{so}^β**):

$$b_i^* = \arg \max_{b_i} \left\{ \sum_{k=1}^{K_{ji}^\beta(b_i, b_{-i})} U_k(\alpha_i) - h^\beta(b_{-i}) \right\} \quad (23)$$

This new auction mechanism improves the efficiency of the POs - SOs interaction and at the same time preserves the necessary properties of the classical optimal auctions.

Proposition 1. *The β -optimal auction mechanism preserves the incentive compatibility and the individual rationality properties of the optimal multi-unit auction introduced in [9].*

The proof of this proposition can be found in our technical report, [23].

C. Efficacy and Requirements of Mechanism \mathcal{M}_R

The pricing that is imposed by the CO, eq. (17), not only bias the channel distribution strategy of the POs, but also changes their bidding strategy. Namely, each PO $j \in \mathcal{M}$ after receiving the bids of his SOs, determines his optimal bid by solving the **PO β -Bidding Problem, (\mathbf{P}_{po}^β)**:

$$r_j^* = \arg \max\{\hat{V}_R(p_j, K_{j0}^\beta, \mathbf{K}_j^\beta) - Q_R(r_j, r_{-j})\} \quad (24)$$

Algorithm 1 (Mechanism \mathcal{M}_R)

1st Stage: Channel Allocation by the CO (β is given).

(1.1:) Each SO i bids to the respective PO, according to problem \mathbf{P}_{so}^β , eq. (23) and informs the CO about his needs (feedback α_i).

(1.2:) Each PO bids to the CO, by solving \mathbf{P}_{po}^β , eq. (24).

(1.3:) The CO solves problem $\mathbf{P}_{co}^{\text{bal}}$, eq. (15)-(16), and allocates $K_{cj}^\beta = K_{j0}^\beta + \sum_{i=1}^N K_{ji}^\beta$ channels to each PO $j \in \mathcal{M}$.

2nd Stage: Channel Redistribution by the POs.

(2.1:) Each PO $j \in \mathcal{M}$ redistributes his channels according to the β -Optimal Allocation Rule, X_j^β .

(2.2:) Each SO i reveals to the CO the allocation decisions of the respective PO (feedback K_{ji}^β).

(2.3:) The CO collects the feedback, calculates the price $L_j(\cdot)$, eq. (17), and determines the overall payment for each PO:

$$\Lambda_j = Y - \beta \left[\sum_{i=1}^N \sum_{k=1}^{K_{ji}^\beta} U_k(\alpha_i) \right] + Q_R(r_j, r_{-j})$$

Parameter $Y \in \mathcal{R}^+$ is a properly selected offset in order to render the payments positive.

The CO determines the channel allocation by solving problem $\mathbf{P}_{co}^{\text{bal}}$, eq. (15) - (16), and calculates the new VCG prices as follows:

$$Q_R = \sum_{m \neq j}^M \hat{V}_R(r_m, \tilde{K}_{m0}^\beta, \tilde{\mathbf{K}}_m^\beta) - \sum_{m \neq j}^M \hat{V}_R(r_m, K_{m0}^\beta, \mathbf{K}_m^\beta) \quad (25)$$

Again, the number of channels allocated to each PO m , $(K_{m0}^\beta, \mathbf{K}_m^\beta)$, depends on bids submitted by all the POs. Also, $(\tilde{K}_{m0}^\beta, \tilde{\mathbf{K}}_m^\beta)$ is the channel allocation when $r_j = 0$. Therefore, the POs are induced to bid truthfully, $r_j^* = p_j$, $j \in \mathcal{M}$. Apparently, \mathcal{M}_R solves both the coordination and the misaligned objectives problem. The improved efficiency of the channel allocation under the β -optimal auction, can be also realized by considering the inequality $U_k(\alpha_i) \geq \pi_k^\beta(b_i) \geq \pi_k(b_i)$ which holds for all the POs and SOs. We summarize mechanism \mathcal{M}_R in Algorithm 1 and Figure 2.

In order to calculate the prices $L_j(\cdot)$, the CO needs to know the actual types of the SOs in the respective secondary market and the amount of spectrum that is allocated to them by the PO. There are many different methods and scenarios about how the CO can acquire this information. First, the SOs may directly provide it through a feedback loop, Figure 1, if there is a trusted relationship among them. Equivalently, the CO may be able to observe the interaction (bidding) of the SOs with the respective PO. Recall that the SOs bid truthfully due to the incentive compatibility property of the β -optimal auction. Finally, the POs may also reveal the outcome of their interactions with the SOs.

Since the controller is on top of this hierarchy and manages the spectrum, we can easily consider many similar methods that will allow him to receive direct or indirect feedback about the SOs - POs interaction. However, in all these cases,

the basic assumption of our mechanism is that the SOs do not anticipate the impact of their bids to the CO decisions (*price takers*). That is, their bidding strategy is not affected by the monitoring/observation by the controller, which is rather expected due to the large number of the SOs, i.e. $N \times M$. Finally, we assume that the SOs bid to the POs before the latter ask for spectrum. If we relax this assumption, the coordination problem is by default unsolvable, but our mechanism still improves the hierarchical spectrum allocation by addressing the objectives misalignment problem. This issue is discussed in the next section, in the context of the dynamic spectrum markets, where it is more prevalent.

V. REGULATION IN DYNAMIC SPECTRUM MARKETS

Until now, we ignored the dynamic aspect of the problem in order to facilitate the analysis and we focused on the novel balanced auction scheme. That is, we implicitly assumed that the interaction of the CO with the POs, and the interactions of the latter with the SOs are performed in the same time scale. This is a realistic assumption since the current suggestions about the spectrum policy reform advocate a more fine grained spatio-temporal management by the regulators, [2]. Nevertheless, the proposed mechanism \mathcal{M}_R can be extended for the case where the CO-POs and POs-SOs interactions are realized in different time scales. Due to lack of space we will briefly explain how the mechanism is adapted for this setting.

Assume that the time is slotted and divided in time periods, $\mathcal{I} = 1, 2, \dots$, where each period is further divided in T time slots, $t = 1, 2, \dots, T$. The CO determines his K_c channels allocation in the beginning of each period while the POs redistribute them in every slot. The CO \mathbf{P}_{co}^{bal} problem for this setting is related to the spectrum allocation for all the T slots within each period:

$$\max_{\{K_{j0}^t, \mathbf{K}_j^t\}} \sum_{t=1}^T \sum_{j=1}^M \left[\hat{V}(p_j, K_{j0}^t, \mathbf{K}_j^t) + \beta \sum_{i=1}^N \sum_{k=1}^{K_{ji}^t} U_k(\alpha_i^t) \right] \quad (26)$$

s.t.

$$\sum_{j=1}^M [K_{j0}^t + \sum_{i=1}^N K_{ji}^t] \leq K_c, \quad t = 1, \dots, T \quad (27)$$

where we have marked with the superscript t the variables that change in each slot. Obviously the CO cannot allocate the spectrum optimally to the POs for the entire period since he is not aware of the future demands of the SOs. Additionally, even if the CO had this information, he could not determine the allocated channels to the POs, K_{cj} , once in each period since these should be adapted to the dynamic secondary demand, K_{ji}^t . Apparently, the **coordination problem** cannot be solved optimally in this setting.

Nevertheless, the CO is still able to solve the **objectives misalignment problem** and induce the POs to allocate their spectrum more efficiently. Assume that the CO-POs interaction is accomplished either without taking into account the secondary demand as in Section III or by considering the average demand of the SOs, $\bar{\alpha}_i$. This will result in a certain suboptimal channel allocation $\bar{K}_c = \{\bar{K}_{cj} : j \in \mathcal{M}\}$. Then, in each slot

as the SOs demand will be realized, they will bid to the POs and at the same time the CO will receive feedback (directly or indirectly) about their needs. This way, the controller will be able to determine the prices $L_j(\cdot)$ for each PO $j \in \mathcal{M}$ at the end of the entire time period:

$$L_j^T(\alpha, \{\mathbf{K}_j^t\}, \beta) = \sum_{t=1}^T \beta \sum_{i=1}^N \sum_{k=1}^{K_{ji}^t} U_k(\alpha_i^t) \quad (28)$$

Obviously, this subsequent pricing at the end of each time period will induce the POs to allocate their spectrum by solving problem \mathbf{P}_{po}^β and maximizing eq. (18), instead of problem \mathbf{P}_{po} , eq. (8)-(9), in each time slot. Therefore, the efficiency loss will be reduced.

VI. SIMULATION RESULTS

In order to obtain insights about the proposed mechanism \mathcal{M}_R , we simulate a representative three-layer hierarchical market with one CO, $M = 2$ POs and $N = 10$ SOs under each PO. We assume that the POs valuation functions for the k^{th} channel are $V_k(p_j) = p_j/k$, where the types p_j are uniformly distributed in the interval $[5, 6]$. Similarly, the SOs valuations are $U_k(\alpha_i) = 0.1\alpha_i/k$, and their types follow a uniform distribution $F(x) = x/4$ on the interval $(0, 4]$. The SOs contributions are $\pi_k(\alpha_i) = (0.2\alpha_i - 0.4)/k$ and the respective β -contributions are $\pi_k^\beta(\alpha_i) = [(0.2 + \beta)\alpha_i - 0.4]/k$. For each random realization of the SOs and POs types, the results are averaged over 40 runs in order to capture the variance on the spectrum demand.

For our study we use as a benchmark the *efficient* channel allocation to the SOs. This allocation corresponds to the hypothetical scenario where the CO would be able to assign directly the channels to both the POs and the SOs and maximize the aggregate spectrum valuations. In the upper plot of Figure 3 we show that in hierarchical unregulated market the number of total channels assigned to the SOs is less than the channels in the *efficient* allocation. Mechanism \mathcal{M}_R with $\beta = 0.1$ reduces this difference and increases the SOs channels. Notice that the number of SOs channels is still less than in the *efficient* allocation scenario, since the goal of the CO is the combined revenue-efficiency balanced allocation.

In the same Figure we show that the number of channels assigned to SOs vary with the value of β . Namely, when $\beta = 0$ the \mathcal{M}_R regulated allocation is identical with the unregulated allocation while for $\beta \approx 0.35$ it reaches the *efficient* allocation. Notice that for larger values of $\beta > 0.37$ the SOs receive even more channels. This means that the CO favors the SOs too much and render the channel allocation inefficient. The impact of β is depicted also in the lower plot of Figure, 4 where we see that for large values the improvement in the aggregate valuation of the POs and SOs becomes negative. For this plot, the number of SOs is $N = 20$ and the system welfare is maximized for $\beta = 0.1$. If β is further increased, the welfare improvement decreases and eventually becomes negative. Finally, we refer the interested readers to [23] for an additional, analytical example of applying \mathcal{M}_R .

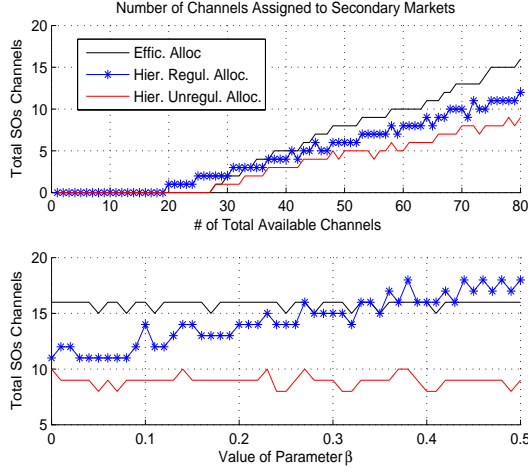


Fig. 3. *Upper Plot:* For $\beta = 0.1$ the regulation mechanism \mathcal{M}_R increases the number of channels assigned to the SOs. *Lower Plot:* The SOs receive more channels for larger values of β ($K_c = 80$).

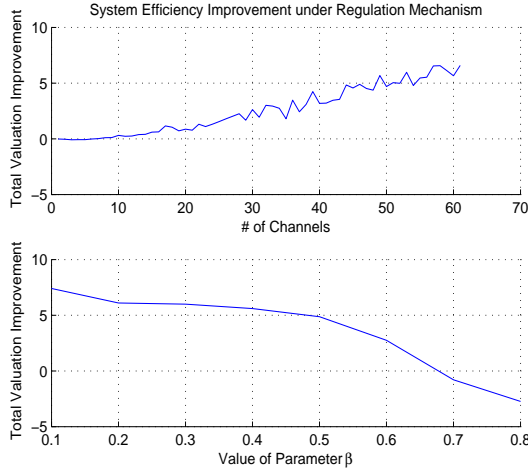


Fig. 4. *Upper Plot:* The aggregate network efficiency (POs and SOs valuations) increases with the \mathcal{M}_R , $\beta = 0.1$, $N = 20$, $K_c = 1 : 60$. *Lower Plot:* For large values of β the network efficiency decreases since the SOs are favored more than the POs.

VII. CONCLUSIONS AND FUTURE WORK

In this paper we proposed an incentive mechanism that can be used by the controller in order to regulate the interaction between the primary and secondary operators in hierarchical spectrum markets and to induce a new market equilibrium. This equilibrium depends on a scalar parameter which is defined by the controller and determines the efficiency of the secondary markets by adjusting the number of channels allocated to the SOs. The mechanism is based on a novel auction scheme which has a revenue-welfare balanced objective. There exist several directions for future work. First, it is interesting to relax the assumption of monopoly markets and study the impact of POs competition on the network performance. We would like to compare the results of competition and regulation in these hierarchical markets. Another intriguing direction is to consider the more realistic setting where there is no prior

knowledge about the SOs types or the family of POs and SOs valuation functions, and apply learning schemes to elicit this hidden information in the spirit of [22]. Finally, even more challenging is to consider the scenario where the SOs anticipate the impact of their bidding to the mechanism and strategize against it in order to gain higher benefits.

APPENDIX

Numerical Example

Consider a market where the CO has 12 channels, there are 2 POs and 2 SOs under each PO. The SOs types are drawn from a uniform cdf $F(x) = x/2$ in the interval $(0, 2]$ and their valuation for the k^{th} channel is $U_k(\alpha) = \frac{\alpha}{k}$. The respective valuations of the POs are $V_k(p) = \frac{3 \cdot p}{k}$. We assume that $p_1 = 1$ with $\alpha_1 = 1.2$ and $\alpha_2 = 1.5$ and $p_2 = 1.2$ with $\alpha_3 = 1.3$ and $\alpha_4 = 1.4$. The contributions of the SOs are $\pi_k(\alpha) = \frac{2\alpha-2}{k}$. If the channel allocation is accomplished with the unregulated hierarchical method then in the first stage the CO allocates the channels to the highest valuations of the POs and these redistribute them comparing their own valuations with the contributions of the SOs in their market. This results in $Ch_{po} = 10$ channels allocated to the POs and $Ch_{so} = 2$ assigned channels to the SOs. If however, the POs were socially aware and considered the valuations of the SOs (instead of their contributions) then the channel allocation would be $[Ch_{po}, Ch_{so}] = [8, 4]$. Finally, even this allocation is not the most efficient because in the first stage the secondary demand has not been considered. If for example the CO was able to allocate directly the channels w.r.t. the POs and SOs valuations, then the allocation would result in $[Ch_{po}, Ch_{so}] = [7, 5]$. Now, assume that we use the proposed mechanism \mathcal{M}_R , with $\beta = 0.2$. In this case, the number of assigned channels will be $Ch_{po} = 9$ and $Ch_{so} = 3$, i.e. increased by 1 for the SOs. Apparently for large values of β the allocation will favor the SOs and the revenue of the POs will decrease.

Proof of Proposition 1

We focus on PO $j \in \mathcal{M}$ with K_{cj} channels. We denote s_{ik} the probability of SO i for receiving the k^{th} channel which depends on the types of all the SOs. Additionally, $c_i(\alpha_i)$ is the payment of each SO i for all the channels he acquired. *Definition 2* and *Lemma 1* in [9] give the necessary conditions for the structure of the bidders (SOs) valuation functions in order to ensure the (IC) and (IR) properties. These conditions hold independently of the objective of the auctioneer (PO) and hence they are not affected by the incorporation of the linear term of the SOs valuation.

The objective of the PO w.r.t. the expected types of the SOs is:

$$E_{\mathcal{A}}[\hat{V}_R(\cdot)] = \sum_{i=1}^N E_{\mathcal{A}}[c_i(\alpha_i)] + \beta \sum_{i=1}^N E_{\mathcal{A}}\left[\sum_{k=1}^{K_{cj}} U_k(\alpha_i) s_{ik}\right] + E_{\mathcal{A}}\left[\sum_{k=1}^{K_{cj}} V_k(p_j) \left(1 - \sum_{i=1}^N s_{ik}\right)\right]$$

The first term is the payment by the SOs, the second is the pricing and the third the valuation for the non-sold channels. After some algebraic manipulations similarly to proof of *Proposition 1* in [9], we get:

$$E_{\mathcal{A}}[\hat{V}_R(\cdot)] = \sum_{i=1}^N E_{\mathcal{A}}\left\{\sum_{k=1}^{K_{c,j}} [(1+\beta)U_k(\alpha_i) - V_k(p_j) - \frac{dU_k(\alpha_i)}{d\alpha} \frac{1-F(\alpha_i)}{f(\alpha_i)}] s_{ik}\right\} - \sum_{i=1}^N \left[\sum_{k=1}^{K_{c,j}} U_k(0) - c_i(0)\right] + E_{\mathcal{A}}\left[\sum_{k=1}^{K_{c,j}} V_k(p_j)\right]$$

Using the necessary (IC) and (IR) conditions from *Lemma 1* in [9], it stems that the β -optimal payment rule is given again by equation (10) of [9]:

$$c_i^*(\alpha_i) = E_{\mathcal{A}}\left\{\sum_{k=1}^{K_{c,j}} U_k(\alpha_i) s_{ik} - \int_0^{\alpha_i} \frac{dU_k(\alpha)}{d\alpha} s_{ik} d\alpha\right\} \quad (29)$$

where the probabilities of allocation are selected so as to maximize the new objective of the auctioneer (instead of revenue only maximization as in [9]). The optimal payment rule is the one that yields zero payment and zero channel allocation for SOs with zero type.

If the problem is *regular* then the payment is as we described in section IV and the first term in the PO's objective is maximized by using the β -optimal allocation rule. This can be easily derived following the proof of the respective *Proposition 2* in [9]. Notice that if the original respective problem in [9] is *regular* then also this modified problem is *regular*. Apparently, the inclusion of the SOs buyers valuations does not affect the monotonicity of the allocation rule nor the critical value property of the payment rule, [19]. Hence, the modified auction is still truthful.

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